

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2) = 0 \Rightarrow 2 \times (-2)^3 - 5 \times (-2)^2 + a \times -2 + a = 0$	M1	3.1a
	Solves linear equation $2a - a = -36 \Rightarrow a =$	dM1	1.1b
	$\Rightarrow a = -36$	A1	1.1b
(3 marks)			
Notes:			
<p>M1: Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$ Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$</p> <p>dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a = .. \Rightarrow a =$</p> <p>A1: $a = -36$</p>			

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$ It should be $\frac{\sin \theta}{\cos \theta} = \tan \theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6^\circ) \neq 2 \sin(-26.6^\circ)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
(3 marks)			
Notes:			
<p>(a)</p> <p>B1: Accept a response of the type 'They use $\frac{\cos \theta}{\sin \theta} = \tan \theta$. This is incorrect as $\frac{\sin \theta}{\cos \theta} = \tan \theta$' It can be implied by a response such as 'They should get $\tan \theta = \frac{1}{2}$ not $\tan \theta = 2$' Accept also statements such as 'it should be $\cot \theta = 2$'</p>			
<p>(b)</p> <p>B1: Accept a response where the candidate shows that -26.6° is not a solution of $\cos \theta = 2 \sin \theta$. This can be shown by, for example, finding both $\cos(-26.6^\circ)$ and $2 \sin(-26.6^\circ)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6^\circ) = +ve$ and $2 \sin(-26.6^\circ) = -ve$ and stating that they therefore cannot be equal.</p> <p>B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5</p>			

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3 \{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
(4 marks)			
Notes:			
M1:	Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$		
A1:	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$		
M1:	Takes out a common factor of $(2x+1)^3$		
A1:	The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$		

Question	Scheme	Marks	AOs
4 (a)	$gf(x) = 3 \ln e^x$	M1	1.1b
	$= 3x, (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$gf(x) = fg(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
(5 marks)			
Notes:			
<p>(a)</p> <p>M1: For applying the functions in the correct order</p> <p>A1: The simplest form is required so it must be $3x$ and not left in the form $3 \ln e^x$ An answer of $3x$ with no working would score both marks</p>			
<p>(b)</p> <p>M1: Allow the candidates to score this mark if they have $e^{3 \ln x} =$ their $3x$</p> <p>M1: For solving their cubic in x and obtaining at least one solution.</p> <p>A1: For either stating that $x = \sqrt{3}$ only as $\ln x$ (or $3 \ln x$) is not defined at $x = 0$ and $-\sqrt{3}$ or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3 \ln x$) is not defined for $x \leq 0$ so therefore there is only one (real) answer. Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)</p>			

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05 \times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{d}{dt}(e^{-0.05t}) = \pm C e^{-0.05t}$	M1	2.1
	$\frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
(4 marks)			
Notes:			
(a)			
M1: Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05 \times 0.5}$			
A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks			
(b)			
M1: Applies the rule $\frac{d}{dt}(e^{kx}) = k e^{kx}$ in this context by stating or using $\frac{d}{dt}(e^{-0.05t}) = \pm C e^{-0.05t}$			
A1: $\frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$			

Question	Scheme	Marks	AOs
6(i)	$x^2 - 6x + 10 = (x - 3)^2 + 1$	M1	2.1
	Deduces "always true" as $(x - 3)^2 \geq 0 \Rightarrow (x - 3)^2 + 1 \geq 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a > 0$ then $ax > b \Rightarrow x > \frac{b}{a}$ if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $= (n + 1)^2 - n^2 = 2n + 1$	M1	3.1a
	Deduces "Always true" as $2n + 1 = (\text{even} + 1) = \text{odd}$	A1	2.2a
		(2)	

(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y = x^2 - 6x + 10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if

$$a < 0 \text{ then } ax > b \Rightarrow x < \frac{b}{a} \text{ or simply } -3x > 6 \Rightarrow x < -2$$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.

For example by attempting $(n + 1)^2 - n^2 = 2n + 1$ or $m^2 - n^2 = (m - n)(m + n)$ with $m = n + 1$

A1: States always true with reason and proof

Accept a proof written in words. For example

If integers are consecutive, one is odd and one is even

When squared odd \times odd = odd and even \times even = even

The difference between odd and even is always odd, hence always true

Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\left(-\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2 + \dots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots \text{ and } k = -\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $ x < 4$, so $x = 1$ can be used	B1	2.4
		(1)	
(5 marks)			
Notes:			
(a)			
M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1 \pm \dots)^{\frac{1}{2}}$			
M1: For an attempt at the binomial expansion with $n = \frac{1}{2}$			
Eg. $(1+ax)^{\frac{1}{2}} = 1 + \frac{1}{2}(ax) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2 + \dots$			
A1: Correct expression inside the bracket $1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots$ which may be left unsimplified			
A1: $\sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots$ and $k = -\frac{1}{64}$			
(b)			
B1: The expansion is valid for $ x < 4$, so $x = 1$ can be used			

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
(7 marks)			
Notes:			
(a) It is important that the student communicates each of these steps clearly			
B1: States the gradient of AB is $-\frac{2}{5}$			
B1: States that y coordinate of $A = 2$			
M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2			
Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$			
A1*: Proceeds to given answer			
(b)			
M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$			
Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2			
M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$			
Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2} '5.8' \times '2'$			
A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$			

Question	Scheme	Marks	AOs	
9	$\int (3x^{0.5} + A) dx = 2x^{1.5} + Ax(+c)$	M1 A1	3.1a 1.1b	
	Uses limits and sets $= 2A^2 \Rightarrow (2 \times 8 + 4A) - (2 \times 1 + A) = 2A^2$	M1	1.1b	
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts $b^2 - 4ac$	M1	1.1b
	$\Rightarrow A = -2, \frac{7}{2}$ and states that there are two roots	States $b^2 - 4ac = 121 > 0$ and hence there are two roots	A1	2.4
(5 marks)				

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5} + Ax(+c)$ where k is a non-zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A dx = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts $b^2 - 4ac$

A1: Either $A = -2, \frac{7}{2}$ and states that there are two roots

Or states $b^2 - 4ac = 121 > 0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_{\infty} = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm \frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
(4 marks)			

Notes:

M1: Substitutes the correct formulae for S_{∞} and S_6 into the given equation $S_{\infty} = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm \frac{1}{\sqrt{2}}$ giving $k = 2$

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geq 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5 = \frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x = 31$	M1	1.1b
	$x = \frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k > 5$ or $k \leq 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5 < k \leq 11\}$	A1	2.5
		(2)	
(6 marks)			
Notes:			
(a)			
B1: $f(x) \geq 5$ Also allow $f(x) \in [5, \infty)$			
(b)			
M1: Deduces that the solution to $f(x) = \frac{1}{2}x + 30$ can be found by solving $-2(3-x)+5 = \frac{1}{2}x+30$			
M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms			
A1: $x = \frac{62}{3}$ only. Do not allow 20.6			
(c)			
M1: Deduces that two distinct roots occurs when $y = k$ intersects $y = f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k > 5$ or $k \leq 11$			
A1: Correct solution only $\{k : k \in \mathbb{R}, 5 < k \leq 11\}$			

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30^\circ = -19.47^\circ$	M1	3.1a
	$\Rightarrow \theta = 5.26^\circ$	A1ft	1.1b
		(2)	
(8 marks)			
Notes:			
(a)			
M1:	Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$		
A1:	$12\sin^2 x + \sin x - 1 = 0$ or exact equivalent		
M1:	Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.		
A1:	$\sin x = \frac{1}{4}, -\frac{1}{3}$		
M1:	Obtains two correct values for their $\sin x = k$		
A1:	All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$		
(b)			
M1:	For setting $2\theta - 30^\circ = \text{their } '-19.47^\circ'$		
A1ft:	$\theta = 5.26^\circ$ but allow a follow through on their $'-19.47^\circ'$		

Question	Scheme	Marks	AOs
13(a)	$R = \sqrt{109}$	B1	1.1b
	$\tan \alpha = \frac{3}{10}$	M1	1.1b
	$\alpha = 16.70^\circ$ so $\sqrt{109} \cos(\theta + 16.70^\circ)$	A1	1.1b
		(3)	
(b)	(i) e.g $H = 11 - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$ or $H = 11 - \sqrt{109} \cos(80t + 16.70)^\circ$	B1	3.3
	(ii) $11 + \sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t + "16.70" = 540$	M1	3.4
	$t = \frac{540 - "16.70"}{80} = (6.54)$	M1	1.1b
	$t = 6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H = 11 - 10 \cos(90t)^\circ + 3 \sin(90t)^\circ$		3.3
		(1)	
(9 marks)			
Notes:			
(a)			
B1: $R = \sqrt{109}$ Do not allow decimal equivalents			
M1: Allow for $\tan \alpha = \pm \frac{3}{10}$			
A1: $\alpha = 16.70^\circ$			
(b)(i)			
B1: see scheme			
(b)(ii)			
B1ft: their 11+ their $\sqrt{109}$ Allow decimals here.			
(c)			
M1: Sets $80t + "16.70" = 540$. Follow through on their 16.70			
M1: Solves their $80t + "16.70" = 540$ correctly to find t			
A1: $t = 6$ mins 32 seconds			
(d)			
B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.			

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \frac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \frac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\frac{dS}{dr}$	M1	3.4
	$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$	A1	1.1b
	Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant	M1	2.1
	Radius = 4.30 cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \frac{500}{\pi r^2} \Rightarrow$ Height = 8.60 cm	A1	1.1b
		(5)	
(c)	States a valid reason such as <ul style="list-style-type: none"> • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans 	B1	3.2a
		(1)	
9 marks			
Notes:			
(a)			
B1: Uses the correct volume formula with $V = 500$. Accept $500 = \pi r^2 h$			
M1: Substitutes $h = \frac{500}{\pi r^2}$ or $rh = \frac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi r h$ to get S as a function of r			
A1*: $S = 2\pi r^2 + \frac{1000}{r}$ Note that this is a given answer.			
(b)			
M1: Differentiates the given S to reach $\frac{dS}{dr} = Ar \pm Br^{-2}$			
A1: $\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$ or exact equivalent			
M1: Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant			
A1: $R =$ awrt 4.30 cm			
A1: $H =$ awrt 8.60 cm			
(c)			
B1: Any valid reason. See scheme for alternatives			

Question	Scheme	Marks	AOs
15	$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) - (6x - 9) dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c) \right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x \right]_0^4 = 2 \times 4^{\frac{5}{2}} - \frac{15}{2} \times 4^2 + 20 \times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
	(10)		
(10 marks)			

Question 15 continued**Notes:**

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) - (6x - 9) dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c) \right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l . See scheme.
- Correct explanation in finding the area of R . In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11 \right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx \right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x \right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R =$ their $36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2} \right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\int \frac{22}{P(11-2P)} dP = \int 1 dt$	B1	3.1a
	Uses (a) and attempts to integrate $\int \frac{2}{P} + \frac{4}{(11-2P)} dP = t + c$	M1	1.1b
	$2 \ln P - 2 \ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2 \ln 9)$	M1	3.1a
	Substitutes $P=2 \Rightarrow t = 2 \ln 2 + 2 \ln 9 - 2 \ln 7$	M1	3.1a
	Time = 1.89 years	A1	3.2a
		(6)	
(c)	Uses ln laws $2 \ln P - 2 \ln(11-2P) = t - 2 \ln 9$ $\Rightarrow \ln\left(\frac{9P}{11-2P}\right) = \frac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\frac{9P}{11-2P}\right) = e^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right) \text{ or } \Rightarrow P = f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \frac{11}{2 + 9e^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
(12 marks)			

Question 16 continued**Notes:****(a)**

B1: Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A$ or B

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$ or equivalent $\frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\int \frac{22}{P(11-2P)} dP = \int 1 dt$ or equivalent

M1: Uses part (a) and $\int \frac{A}{P} + \frac{B}{(11-2P)} dP = A \ln P \pm C \ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg
 $2 \ln P - 2 \ln(11-2P) = t + c$

M1: Substitutes $t=0$ and $P=1$ to find c

M1: Substitutes $P=2$ to find t . This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2 \ln P - 2 \ln(11-2P) = t + c$ to $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $e^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d \Rightarrow \frac{P}{11-2P} = e^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $e^{-\frac{1}{2}t}$ For example

$\frac{P}{11-2P} = e^{\frac{1}{2}t+d} \Rightarrow \frac{11-2P}{P} = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} - 2 = e^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} = 2 + e^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P = \frac{11}{2+9e^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$ oe